ANR ASCETE : Partenaire Grenoble

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2. Chirp-Based Ridge Detection
3. Rényi entropy and optimal window size computation
4. Multicomponent signal denoising based on linear chirp approximation
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6. Post-doc work: Neha Singh (Feb 2020-)
7. Inter-partners work

Introduction

The analysis of multicomponent signals at the heart of signal processing research for over 60 years. A challenge is that of dealing with signals with multiple AM-FM components

$$x(t) = \sum_{p=1}^{P} x_p(t), \text{ with } x_p(t) = A_p(t) e^{2i\pi\phi_p(t)}$$

with $A_p(t) > 0$ and $\phi'_p(t) > 0$, sought instantaneous amplitude and frequency

Chirp-Based Ridge Detection [Colominas et al.'19]

Let us consider the linear chirp $x(t) = e^{i2\pi(at+bt^2)}$. Its STFT reads:

$$F_x^g(t,f) = x(t)\int_{-\infty}^{+\infty} g(u)e^{i2\pi bu^2}e^{-i2\pi(f-(a+2bt))u}du \\ = x(t)\widehat{g_{\phi''}}(f-\phi'(t)), \tag{1}$$

with $g_{\phi''}(t) = g(t)e^{i2\pi\frac{\phi''}{2}t^2}$, $\phi'(t)$ being the IF of x and $\phi''(t) = 2b$ its instantaneous CR.
When $g(t) = e^{-\sigma t^2}$, Eq. (1) can be rewritten as:

$$F_x^g(t,f) = x(t)\sqrt{\frac{\pi}{\sigma - i\pi\phi''}}e^{\frac{-\sigma\pi^2(f-\phi'(t))^2}{\sigma^2+\pi^2\phi''^2}}e^{\frac{-i\phi''\pi^3(f-\phi'(t))^2}{\sigma^2+\pi^2\phi''^2}}. \tag{2}$$

[Colominas et al.'19] M. Colominas, S. Meignen and D-H. Pham, "Time-Frequency Filtering Based on Model Fitting in the Time-Frequency Plane", IEEE Signal Processing Letters, vol. 26, no. 5, pp. 660-664, 2019

New IF estimation based on STFT modulus approximation

- We can then think of more complicated signals $x(t) = A(t)e^{i2\pi\phi(t)}$, which, however, admits *locally* a linear chirp approximation.
- We define, for every t, a model with two free parameters:

$$\rho(f, \tilde{\phi}'(t), \tilde{\phi}''(t)) = \sqrt{\frac{\pi}{\sqrt{\sigma^2 + \pi^2 \tilde{\phi}''(t)^2}}} e^{\frac{-\sigma\pi^2(f-\tilde{\phi}'(t))^2}{\sigma^2+\pi^2\tilde{\phi}''(t)^2}}, \qquad (3)$$

and estimate the IFs of a given MCS by solving:

$$\max_{\Phi} \sum_{l=1}^{L} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |F_x^g(t,f)| \rho(f, \tilde{\phi}_l'(t), \tilde{\phi}_l''(t)) df dt, \qquad (4)$$

where $\Phi := (\tilde{\phi}_l', \tilde{\phi}_l'')_{l=1,\ldots,L}$.

Analysis

- If $x(t) = e^{i2\pi(at+bt^2)}$, then $\nabla_{\tilde{\phi}',\tilde{\phi}''}\langle |F_x^g(t,f)|, \rho(f, \tilde{\phi}'(t), \tilde{\phi}''(t))\rangle = 0$, iff $\tilde{\phi}'(t) = a + 2bt$ and $\tilde{\phi}''(t) = 2b$.
- The unique solution to (4) using an optimization methods without derivatives, e.g. the golden-section search, in which each variable is independently treated.
- Two steps strategy: first one seeks the optimum IF, and then the optimum absolute value of C.
  $\frac{\partial}{\partial\tilde{\phi}'}\langle |F_x^g(t,f)|, \rho(f, \tilde{\phi}', \tilde{\phi}'')\rangle_{\tilde{\phi}'=a+2bt} = 0$ regardless the value of $\tilde{\phi}''$.

Discrete-time implementation

In a discrete-time framework, one can transpose the above analysis to that of a discrete-time signal $x[n] = e^{i2\pi(an+bn^2)}$ as:

$$F_x^g[n,k] \approx x[n]\sqrt{\frac{\pi}{\sigma - i\pi\phi''}}\, e^{\frac{-\sigma\pi^2(k\Delta f - \phi'[n])^2}{\sigma^2+\pi^2\phi''^2}}\, e^{\frac{-i\phi''\pi^3(k\Delta f - \phi'[n])^2}{\sigma^2+\pi^2\phi''^2}}, \qquad (5)$$

where $\phi'[n] = a + 2bn$ is the real-valued IF of $x[n]$ and $\phi'' = 2b$ its CR. Then, a discrete-time version of (4) is implemented:

$$\max_{\Phi} \sum_{l=1}^{L}\sum_{n=1}^{N}\sum_{k=0}^{K-1} |F_x^g[n,k]|\rho(k\Delta f, \tilde{\phi}_l'[n], \tilde{\phi}_l''[n]), \qquad (6)$$

where

$$\rho(k\Delta f, \tilde{\phi}_l'[n], \tilde{\phi}_l''[n]) = \sqrt{\frac{\pi}{\sqrt{\sigma^2 + \pi^2\tilde{\phi}''[n]^2}}\, e^{\frac{-\sigma\pi^2(k\Delta f - \tilde{\phi}'[n])^2}{\sigma^2+\pi^2\tilde{\phi}''[n]^2}}}. \qquad (7)$$

Eq. (6) offers the possibility to estimate the IFs as real-valued functions.

Mode reconstruction

Once the IF and the absolute value of the CR are computed, the modulus of the STFT can be estimated by:

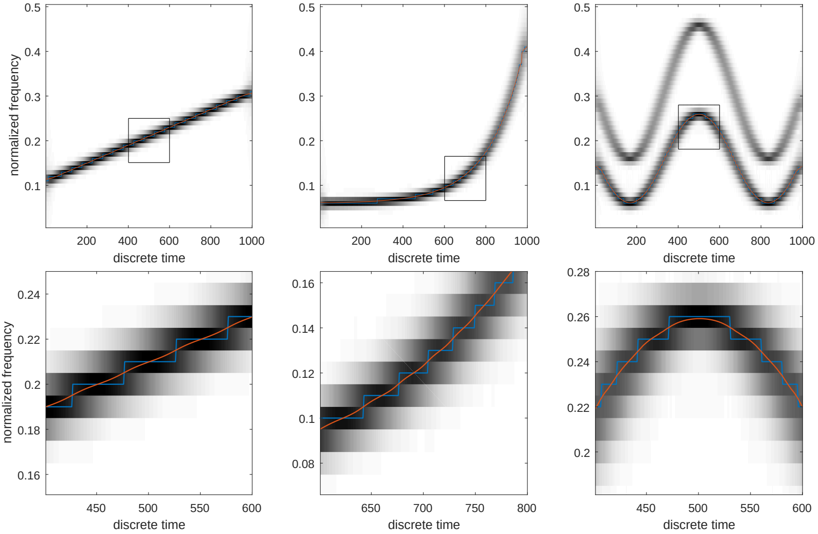
$$|F_x^g[n,k]_{est}| = \frac{\langle |F_x^g[n,k]|, \rho(k\Delta f, \tilde{\phi}'[n], \tilde{\phi}''[n]) \rangle}{\|\rho(k\Delta f, \tilde{\phi}'[n], \tilde{\phi}''[n])\|_2^2} \rho(k\Delta f, \tilde{\phi}'[n] \tilde{\phi}''[n]). \quad (8)$$

The STFT of the mode is approximated by

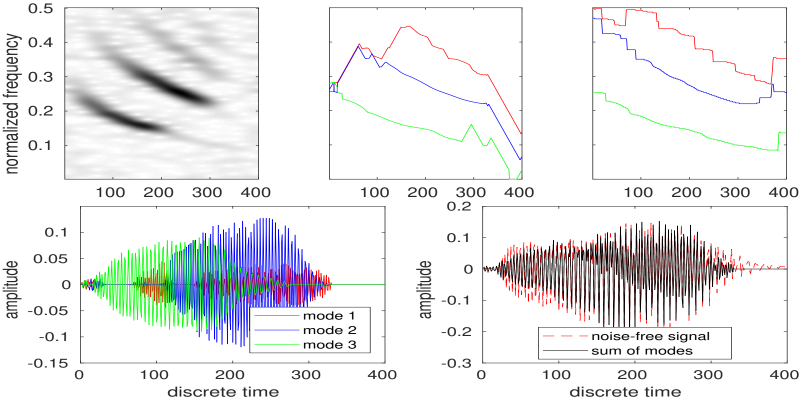
$$F_{x_l}^g[n,k]_{est} = \left| F_{x_l}^g[n,k]_{est} \right| e^{i \arg F_x^g[n,k]}. \quad (9)$$

Finally, the reconstructed mode is obtained by inverting the STFT in the vicinity of the ridge.

Illustrations



Illustrations



Rényi entropy computed on the magnitude of the STFT, defined in the continuous time and frequency context on $R = [0, 1] \times [0, L]$:

$$H^R_{\alpha,g}(x) := \frac{1}{1-\alpha} \log_2 \left(\int_R \left(\frac{|F^g_x(t,\xi)|}{\int_R |F^g_x(t',\xi')| dt' d\xi'} \right)^\alpha dt d\xi \right).$$

Discretizing R one obtains a grid G, and a discretize version of the Rényi entropy:

$$H^G_{\alpha,g}[x] := \frac{1}{1-\alpha} \log_2 \left(\sum_{(n,k)\in G} \left(\frac{|F^g_x[n,k]|}{\sum_{(n',k')\in G} |F^g_x[n',k']|} \right)^\alpha \right) - \log_2(N).$$

[Meignen et al.'20], S. Meignen, M. Colominas and D-H. Pham, "On the Use of Rényi Entropy for Optimal Window Size Computation in the Short-Time Fourier Transform", ICASSP 2020

Behaviour of the Rényi entropy on a pure tone signal

- The first study considers a pure tone signal $x(t) = e^{2i\pi\omega_1 t}$, for $t \in [0,1]$ and assume $g(t) := g_s(t) = e^{-\pi\frac{t^2}{\sigma_s^2}}$. One can show that:

$$H^R_{\alpha,g_s}(x) \approx H^R_{\alpha,g_1}(x) - \log_2(\sigma_s),$$

  provided, when $\sigma_s \leq 1$, $e^{-\pi\sigma_s^2\omega_1^2} \leq \varepsilon$, meaning $\frac{\sqrt{-\frac{\log(\varepsilon)}{\pi}}}{\omega_1} \leq \sigma_s$.
- The Gaussian window being truncated, upper bound $\sigma_{\max}$ on σ_s imposed, i.e. $e^{-\pi\frac{\lfloor\frac{N-1}{2}\rfloor^2}{\sigma_{\max}^2 L^2}} = \varepsilon$, meaning $\sigma_s \leq \sigma_{\max} = \frac{\lfloor\frac{N-1}{2}\rfloor}{L}\sqrt{\frac{-\pi}{\log(\varepsilon)}}$.

$\Rightarrow$ We thus determine an interval of interest in which the Rényi entropy decreases with respect to σ_s.

Extension to several pure tone modes

For two Gaussian logons, slightly interfering in the TF plane one has:

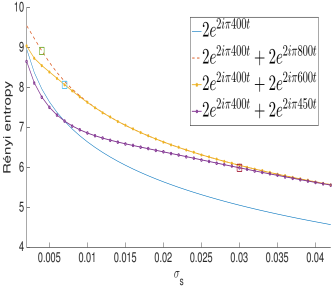
$$H^R_{\alpha,g_s}(x_1 + x_2) \approx H^R_{\alpha,g_s}(x_1) + 1,$$

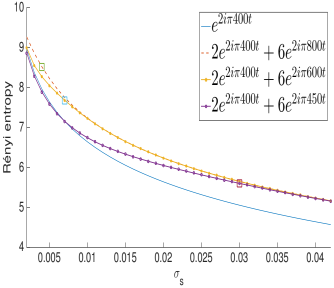
For a signal made of two pure tones, the separation condition reads:

$$\frac{\sqrt{-\frac{\log(\varepsilon)}{\pi}}}{\min(\omega_1, \frac{\omega_2 - \omega_1}{2})} \leq \sigma_s.$$

More generally, for a sum of K pure tones $\frac{\sqrt{-\frac{\log(\varepsilon)}{\pi}}}{\min(\omega_1, 2\Delta)} \leq \sigma_s$.

Illustrations





Case of linear chirps

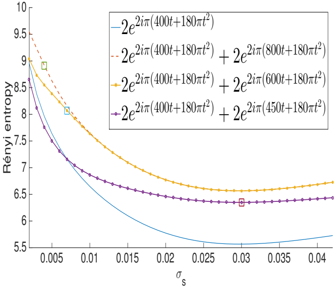
- Similarly to what we had in the pure tone signal case, we have

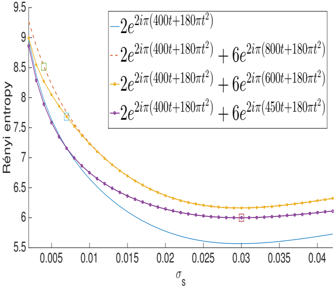
$$H^R_{\alpha,g_s}(x) \approx H^R_{\alpha,g_1}(x) - \log_2(\tilde{\sigma}_s),$$

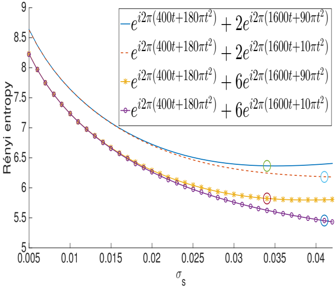
if, $\frac{\sqrt{-\frac{\log(\varepsilon)}{\pi}}}{\phi'(t)} \leq \frac{\sigma_s}{\sqrt{1+\sigma_s^4\phi''(t)^2}} \leq \sigma_{\max}$.

- The minimum of $\frac{\sigma_s}{\sqrt{1+\sigma_s^4\phi''(t)^2}}$ attained for $\sigma_s = \frac{1}{\sqrt{\phi''(t)}}$.
- For two parallel linear chirps, lower bound replaced by $\frac{\sqrt{-\frac{\log(\varepsilon)}{\pi}}}{\min(\phi_1'(t),\phi_2'(t)-\phi_1'(t))}$, and the minimum located at $\sigma_s = \frac{1}{\sqrt{\phi''(t)}}$
- For two non parallel linear chirps, when the minimum of the Rényi entropy exists it is attained for $\sigma_s = \frac{1}{\sqrt{\frac{\phi_1''(t)+\phi_2''(t)}{2}}}$ (conjecture).

Illustrations







- if x is the linear chirp $x(t) = Ae^{2i\pi(at+b\frac{t^2}{2})}$, it STFT reads

$$F_x^g(t,\eta) = F_x^g(t, a+bt)e^{\frac{-\pi\sigma^2(1+ib\sigma^2)}{1+(b\sigma^2)^2}(\eta-a-bt)^2}$$

One has the following reconstruction formula:

$x(t) = \frac{F_x^g(t,a+bt)}{g(0)} \int_{\mathbb{R}} e^{\frac{-\pi\sigma^2(1+ib\sigma^2)}{1+(b\sigma^2)^2}(\eta-a-bt)^2} d\eta,$

- Instantaneous frequency (IF) $a+bt$ and *chirp rate* (CR) b are to be estimated. We consider the following complex modulation operator:

$$\tilde{q}_x(t,\eta) = \frac{1}{2i\pi} \frac{F_x^{g''}(t,\eta)F_x^g(t,\eta) - \left(F_x^{g'}(t,\eta)\right)^2}{F_x^{tg}(t,\eta)F_x^{g'}(t,\eta) - F_x^{tg'}(t,\eta)F_x^g(t,\eta)},$$

which is such that $\widehat{q}_x(t,\eta) = \Re\{\tilde{q}_x(t,\eta)\} = b$. Then, introducing $\tilde{\omega}_x(t,\eta) = \eta - \frac{1}{2i\pi}\frac{F_x^{g'}(t,\eta)}{F_x^g(t,\eta)}$ and $\tau(t,\eta) = -\frac{F_x^{tg}(t,\eta)}{F_x^g(t,\eta)}$, one defines:

$$\widehat{\omega}_x^{[2]}(t,\eta) = \Re\{\tilde{\omega}_x(t,\eta) - \tilde{q}_x(t,\eta)\tau(t,\eta)\},$$

which equals $a+bt$ when x is a linear chirp.

Practical implementation

- When x departs from a linear chirp, the STFT approximated by:

$$F_x^g(t,\eta) = F_x^g(t,\widehat{\omega}_f^{[2]}(t,\eta))e^{\frac{-\pi\sigma^2(1+i\hat{q}_f(t,\eta)\sigma^2)}{1+(\hat{q}_f(t,\eta)\sigma^2)^2}(\eta-\widehat{\omega}_f^{[2]}(t,\eta))^2}.$$

- In the discrete time case, assuming $\tilde{x}$ is the sum of noisy modes, $(\tilde{\varphi}_p)_{p=1,\cdots,P}$ the ridges extracted from $F_{\tilde{x}}^g$, defining $\tilde{\psi}'_p[m] := \widehat{\omega}_{\tilde{x}}^{[2]}[m,\tilde{\varphi}_p[m]]$, and $\tilde{\psi}''_p[m] := \widehat{q}_{\tilde{x}}[m,\tilde{\varphi}_p[m]]$, we may write:

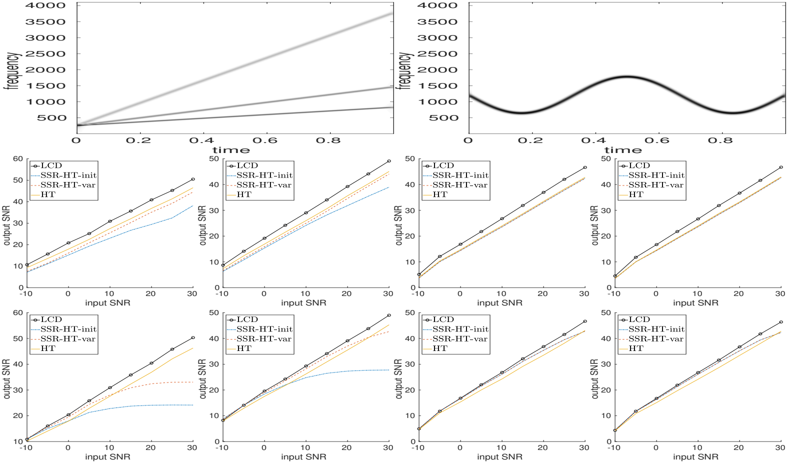
$$F_{x_p}^g(\frac{m}{L},k) \approx \tilde{F}_{x_p}^g[m,k] := F_{\tilde{x}}^g\left(\frac{m}{L},\tilde{\psi}'_p[m]\right)e^{-\frac{\pi\sigma^2(1+i\tilde{\psi}''_p[m]\sigma^2)}{1+(\tilde{\psi}''_p[m]\sigma^2)^2}(k\frac{L}{N}-\tilde{\psi}'_p[m])^2}.$$

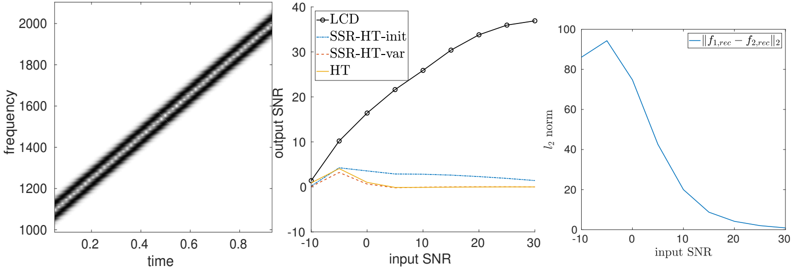
- But $F_{\tilde{x}}^g\left(\frac{m}{L},\tilde{\psi}'_p[m]\right)$ unknown, considering $k_0 := \lfloor\tilde{\psi}'_p[m]\frac{N}{L}\rceil$, one obtains:

$$F_{\tilde{x}}^g(\frac{m}{L},\tilde{\psi}'_p[m]) = F_{\tilde{x}}^g[m,k_0]e^{\frac{\pi\sigma^2(1+i\tilde{\psi}''_p[m]\sigma^2)}{1+(\tilde{\psi}''_p[m]\sigma^2)^2}(k_0\frac{N}{L}-\tilde{\psi}'_p[m])^2}.$$

- Reconstruction performed through: $f_p[m] \approx \frac{1}{g(0)}\sum_{k=0}^{N-1}\tilde{F}_{x_p}^g[m,k]$.

Illustrations




frequency
time
2000
1800
1600
1400
1200
1000
0.2
0.4
0.6
0.8
output SNR
input SNR
LCD
SSR-HT-init
SSR-HT-var
HT
40
30
20
10
0
-10
-10
0
10
20
30
l_2 norm
input SNR
$\|f_{1,rec} - f_{2,rec}\|_2$
100
80
60
40
20
0
-10
0
10
20
30

PhD thesis work: Nils Laurent (Oct 2019-)

- Mode reconstruction based on linear chirp estimation (submitted paper).
- New ridge extraction technique (Task 1.2)
- Multivariate Synchrosqueezing transform and applications to the study of ECG and EEG signal.
- SST-Based NMF (Task 3.1 relation with Roland)

Post-doc work: Neha Singh (Feb 2020-)

- How synchrosqueezing deals with noise (Task 1.1, recent paper by H-T. Wu)
- Ridge extraction and colliding modes (Task 1.2)
- Choice of optimal window in TF representation (thesis subject, recent development in adaptive synchrosqueezing transform).

Inter-partners work

- Connection between ridge extraction technique and mode extraction using DNN (Task 1.2 and 2.3, Grenoble, Nantes and Paris)
- Phase retrieval in SST, relation with NMF (Task 3.3 in relation with Roland)
- Phase retrieval from STFT (Task 3.4 in relation with Roland)