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A Novel Pseudo-Bayesian Approach for Robust Multi-Ridge Detection and Mode Retrieval

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Introduction

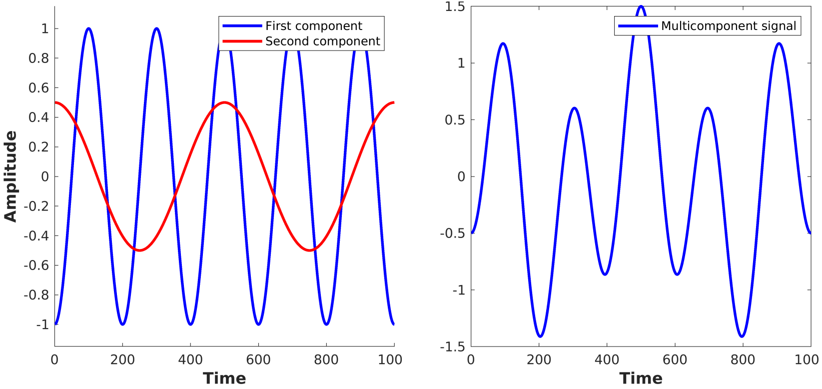
- Focus on multi-component signals (MCS).

$$x(t) = \sum_{k=1}^{K} x_k(t) \quad , \text{with } x_k(t) = a_k(t)\, e^{j\phi_k(t)}, \tag{1}$$

Investigated approaches

- Mixture of K superimposed components.
- $a_k(t)$ and $\phi_k(t)$ the time-varying amplitude and frequency of component k.
- Amplitude and frequency modulated.

Introduction



Motivation

- Variety of application.
- Audio, medical, astronomical, echolocation,...

Introduction

Context of this work

- Time-frequency representation (TFR).
- Well design representation for our objective.
- Allows to observe the instantaneous frequency (IF) trajectory of each mode as a ridge.

Subject of research: ASCETE project

- New ridge extraction technique.
- Modes extraction and demodulation.

Challenge

- Large variety of real signals.
- Need for a general extraction approach.
- Presence of external spurious noise.

Plan

1. Observation model
2. Estimation strategy
3. Results
4. Conclusion

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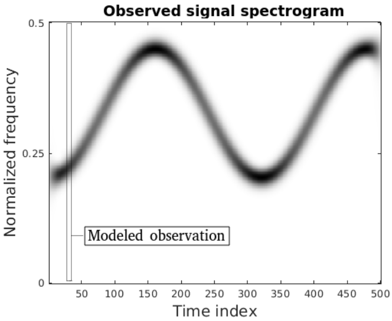
Observation model

Our model

$$p(z|\bar{m}_n) = g\left(z - \bar{m}_n\right),$$

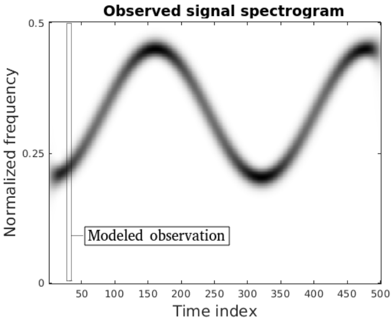


- z : frequency in $[0, M-1]$.
- $\bar{m}_n$: ridge position in the n-th time bin.
- $g(m) = \frac{2\sqrt{\pi}L}{M}\, e^{-\left(\frac{2\pi mL}{M}\right)^2}$

Observation model

Our model

$$p(z|\bar{m}_n) = g\left(z - \bar{m}_n\right), \tag{2}$$



Limitations

- Simple model.
- Computationally attractive.
- Presence of noise neglected.
- Assumes for the presence of a single component.

Observation model

Limitations

- Lack of generality of the postulated model.
- Discrepancies with noisy observations.
- Or in the presence of multiple components.
- Inefficiency of maximum likelihood estimation (MLE).

Proposed approach

- Estimation performance does not only depend on the model quality.
- Modification of the similarity measure[1].

[1] Q. Legros, S. McLaughlin, Y. Altmann, S. Meignen and M. E. Davies. Robust depth imaging in adverse scenarios using single-photon Lidar and beta-divergences, 2020.

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Estimation strategy

Note that

- Performing MLE $\Leftrightarrow$ minimizing Kullback-Leibler(KL) divergence between model and observations.
- KL divergence not suitable when the postulated model is inaccurate.
- Implies model mismatch.
  - In the presence of external spurious noise.
  - When observing multicomponent signals.

Alternative variational objective

- KL divergence replaced by the Rényi and β divergences.
- Allow respectively for mode seeking character and robustness.

Estimation strategy

Alternative inference

- Variational inference based on alternative divergences.
- Need for the divergences cross entropy[2].

Cross entropy

- For the β-divergence (β-d), $\beta > 0$

$$\mathrm{CE}_\beta(\bar{m}_n) = -\frac{1+\beta}{\beta} \sum_m p(s_{n,m}|\bar{m}_n)^\beta + \int p(z|\bar{m}_n)^{1+\beta} \mathrm{d}z. \tag{3}$$

- For the Rényi divergence (R-d), $\alpha > 0, \alpha \neq 1$

$$\mathrm{CE}_\alpha(\bar{m}_n) = \frac{1}{\alpha - 1} \log \left(\sum_m s_{n,m}^\alpha p(s_{n,m}|\bar{m}_n)^{1+\alpha} \right). \tag{4}$$

[2] F. Futami, I. Sato, M. Sugiyama. Variational Inference based on Robust Divergences, 2018.

Estimation strategy

Pseudo-Bayesian estimation

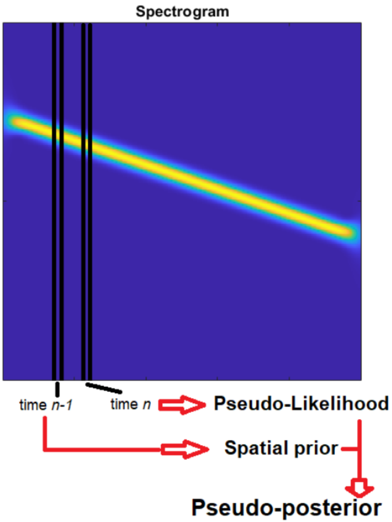
- Approximate posterior distribution obtained by maximizing the evidence lower-bound (ELBO)[2]

$$p(\bar{m}_n|\boldsymbol{s}_n) \propto e^{-MCE(\bar{m}_n)} p(\bar{m}_n). \tag{5}$$

- Spatial prior model $p(\bar{m}_n)$ discussed hereafter.
- Plug cross entropy for alternative objectives.
- Ridge position estimated by minimum mean squared error(MMSE).

[2] F. Futami, I. Sato, M. Sugiyama. Variational Inference based on Robust Divergences, 2018.

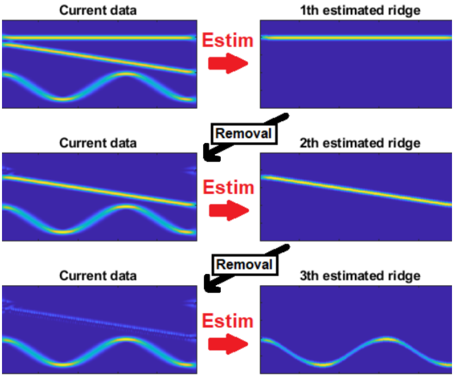
Estimation strategy



Online estimation

- Ridge extraction.
- Iterating on time axis.
- Sequential propagation of the information.
- Spatial prior : Gaussian random walk.
- Complexity: Variational inference.
- Accuracy: Backward correction.

Estimation strategy



Sequential demodulation

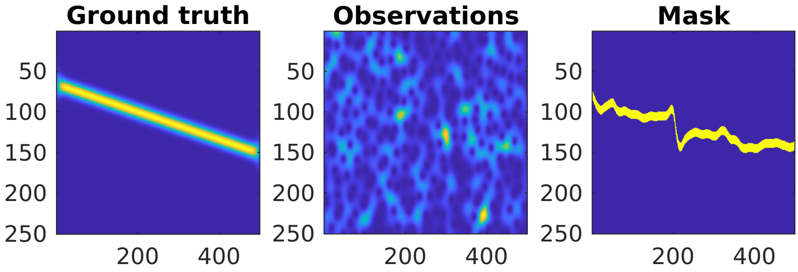
- Extraction of a ridge.
- Update of the data.
- Removing energy associated with the estimated component.
- Stopping criterion: method of choice[3].

[3] V. Sucic and N. Saulig and B. Boashash. Estimating the number of components of a multicomponent nonstationary signal using the short-term time-frequency Rényi entropy, 2011.

Results - examples 1

Numerical experiments

- Single linear chirp component.
- SNR = -15dB.
- Rényi divergence, $\alpha = 0.2$.



Results - examples 2

Numerical experiments

- Single linear chirp component.
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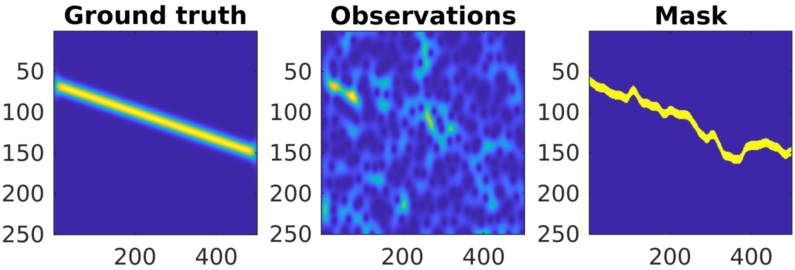

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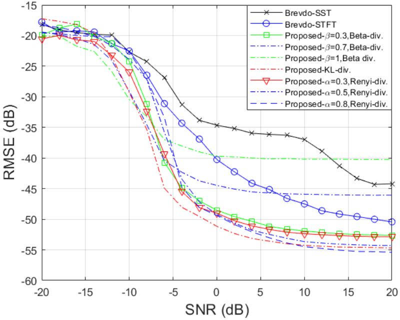
Results

Numerical experiments

- Reconstruction of a single sinusoidal component merged in a with Gaussian noise.
- Comparison to Brevdo approach[4]



- Comparison using several variational objectives.
- RMSE: Relative mean square error (in dB).
- Better estimation using the proposed method.
- Distinct working regions.

[4] E. Brevdo, N. S. Fuckar, G. Thakur and H-T Wu. The synchrosqueezing algorithm: a robust analysis tool for signals with time-varying spectrum, 2011.

Results

Numerical experiments

- Reconstruction performance of a MCS.
- Reconstruction quality factor: $\mathrm{RQF} = 10\log_{10}\left(\frac{||x||^2}{||x-\hat{x}||^2}\right)$.
- Assessment: Component-wise RQF.

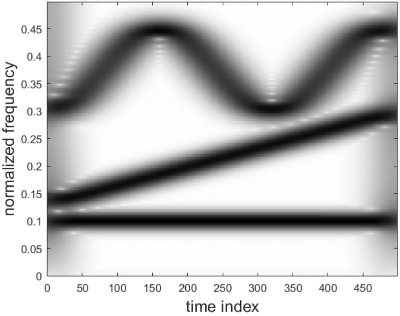


Figure: Spectrogram of the analyzed multicomponent signal.

Three components

- Sinusoidal frequency modulated (FM).
- Linear chirp.
- Sinusoid.

Results

Table: RQF of each components (averaged over 100 realizations) for the different competing approaches for a SNR = 10 dB.

	Sinusoid	Linear chirp	Sin. FM chirp	Average
Brevdo	16.10	**15.46**	2.86	11.47
Brevdo-Synchrosqueezing	16.43	15.34	5.24	12.34
Proposed β-d, $\beta = 0.5$	**16.71**	15.22	9.13	13.69
Proposed β-d, $\beta = 0.8$	16.45	14.92	5.49	12.29
Proposed-KL	2.46	2.65	1.18	2.10
Proposed R-d, $\alpha = 0.5$	16.59	15.24	**9.57**	**13.80**
Proposed R-d, $\alpha = 0.8$	15.44	15.22	7.84	12.83

Numerical experiments

- Our method obtains the best averaged RQF using R-d ($\alpha = 0.5$).
- Efficient recovery of the sinusoidally FM chirp.
- Alternative divergences circumvent the lack of generality of our model.

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Conclusions and perspectives

Conclusions

- A novel pseudo-Bayesian estimation procedure to demodulate MCS.
- An adaptive approach accounting to the presence of arbitrary external noise.
- Efficient extraction of modulated frequency components.

Future works

- Consideration of overlapping ridges.
- Generalizing the variational objective ($\alpha\beta$-divergence).
- Estimation of divergences hyperparameters.

Thanks for your attention !

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